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THE GRADUAL LINEARIZATION OF GERMAN GEOMETRY TEACHING

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For teachers, it can be worthwhile to learn from history how in the 19th century mathematics as a science inspired mathematics teaching. "Neuere Mathematik", "Meraner Reform" and "Anschauungslehre" established a close connection between scientific mathematics and the exemplified elementary school mathematics. They initiated fruitful discourses stimulating the development of teachers, restructuring the school and the university system and leading to new curricula. Shall we reintroduce conic sections in different approaches, contexts and with applications?

Student teachers as prospective actors in curriculum development

How did the current curriculum of secondary school mathematics come about? Over the past millennia, a large number of (still-valid) mathematical truths have been discovered, discussed, and proven. Who chooses what to teach in class and on what grounds is such a decision taken? To what extent does the development of modern mathematics effect curriculum development? Students obtaining a teaching profession should ask themselves these questions also in order to recognize their own prospective responsibility in mathematics curriculum development. The selection of mathematical subjects and their presentation in school mathematics is also made in the class and can possibly mean to put the current curriculum into question. The appearance and the disappearance of conic sections and descriptive geometry in mathematics teaching offer ample opportunities to mathematics teachers to look into their own traditions, as for instance the tradition of pre-university teaching in secondary school, and into the interrelation between developments in mathematics and its teaching.

Scientific development and the elementarization of mathematics

This contradiction between mathematics as a science and the exemplified elementary school mathematics was the basis of different fruitful discourses in the 19th century, which led to a transformation of the pre-university teaching that was to great extent shaped by Euclid's elements until then. The occupation with the development of geometry teaching in the 19th century is especially illuminative since here the scientific development and the elementarization of mathematics happened parallel to each other at frequent intervals and often by the same people. Max Simon for instance notes: "When you look at the elementary geometry of the 19th century, it is especially worth mentioning, how the great developments of science also come to light in elementary geometry." (Simon, 2011, translation by the authors). These developments are among others descriptive geometry (Monge), Analysis situs (Carnot, von Staudt), geometrical constructions (Steiner), projective properties (Poncelet), barycentric coordinates (Möbius), linear algebra and algebra (Graßmann, Plücker), analytical geometry (Gergonne).

Flourishing teaching culture in *Neuere Geometrie*

Under the banner of *Neuere Geometry* (Newer Geometry), the research in geometry as a science and the educational reforms in mathematics teaching merge. "The New Geometry, seen from its genesis, is not as much in contrast to the geometry of the ancient than it is in contrast to analytical geometry… Analytical geometry as a subject is a continuation to the elements, but as a method, it is in contrast to the elements "(Pasch, 1882, S.1, translation by the authors). The immediate junction of new developments in mathematics with teaching reforms is also fostered by the professionalization of the teachers, restructuring of the school system, development of new curricula as well as changes in the university system. In 1810, for instance, the examination of teachers for *higher schools* was introduced, which did not only require decent knowledge in philosophy and history but also in mathematics. In 1812, the deep knowledge of Euclid's books 1-6, 11 und 12 became a general requirement for the final examination (*Abiturprüfung*) at school.

With the so called *Süvernscher Normalplan* (1816) and a renewed lesson scheme for mathematics classes, for instance the analytical approach to conic sections became a teaching subject in grade 10 and 11 (*Sekunda*, 16-17 years old) at the *Gymnasium* (pre-university secondary school). While the conic sections were taken up in the curriculum, text books with different approaches to the subject appeared. For an impression about these different presentations, we recommend a look at the antiquarian or digitally available text books of this time. The theologist Johann Andreas Matthias (1813) for instance, chose the approach to conic sections along the Apollonian way. The mathematician Johann August Grunert (1824) however, used the analytical method to deal with conic sections in his teaching script with exercises and their demonstrated solutions. Also, the mathematician, philosopher, reform educator (*Reformpädagoge*), politician, school teacher (*Schulmann*) and founder of the Berlin Pedagogical Seminar, Karl Heinrich Schellbach, composed in 1843 a text book about conic sections, that was published by Max Simon. An impression of later teaching texts on the subject, which even took projective approaches into account, as well as a detailed analysis of the presentation of *Neuere Geometrie* is provided by Sebastian Kitz in his dissertation on *Neuere Geometrie* as teaching subject for higher teaching institutions (*höhere Lehranstalten*) between 1870 and 1920 (Kitz, 2015). Examples of the appearance of modern mathematical developments in elementary geometry, as it was described by Max Simon, are also Hermann Hankel's (Hankel, 1875) and Jakob Steiner's (Steiner, 1876) synthetical treatises on conic sections.

The royal road to geometry

The expectations regarding the reforming power of *Neuere Geometrie* become apparent in Hankel's way to rephrase the well-known ancient anecdote: "There is no royal road to geometry. We, however, can add: The *Neuere Geometrie* is this royal road." (Hankel 1875, S.33, translation by the authors). Despite these high expectations in the *Neuere Geometrie* and its rapid development as scientific discipline, the school reform initially experienced setbacks. The decision between synthetic and analytic geometry, between Euclidean and *Neuere Geometrie* was at first in the secondary school (Gymnasia) taken in favor of Euclidean geometry without the treatment of conic sections. Consequently in 1837, by a Prussian circular directive (*Preußisches Zirkularreskript, i.e. Runderlass*) of Johann Schulze, the successor of Süvern, disposed a reduction of scheduled mathematics lessons

and the removal of conic sections of the curriculum at the Gymnasium. More details and examples for the implementation of the curricula of Süvern and Schulz can be found in particular in the study on the history of the *Ratsgymnasium Bielefeld* (Biermann, 2010) and in the dissertation of Martina Strub (2008, S. 67 ff). The latter sheds light on the then popular quote that was ascribed to the classical philologist Johannes Schulze: "… in one line of Cornelius Nepos, there is more formative power than in twenty mathematical formulas."

Meraner Reform and *Anschauungslehre*

Only during the gathering of philologists in 1864 in Jena, it came to the foundation of a mathematical-pedagogical section and to the revival of the discussion on conic sections for the teaching at secondary school. In these discussions, the treatment of conic sections in analytical form was linked with the notions of variable and function and hence with the intentions of the *Meraner Reform* for the introduction of differential- and integral calculus (Schimmack, 1911). The proposals of the *Meraner Reform* did not only take those parts of the theory of conic sections with a direct relation to the notion of function into account, but also recommended to deal with conic sections in analytical and synthetical form – even with application to the elements of astronomy, albeit without exemplification of its implementation. Another source of the reformation of the Euclidean tradition of geometry teaching is the development of the *Anschauungslehre*, an education to an inner intuition and view. The geometry book in three volumes of Henrici and Treutlein (Henrici & Treutlein, 1981-1983) as well as Treutlein's *Anschauungslehre* (Treutlein, 1911) – called by Felix Klein "exceptionally noteworthy book" (Klein, 2016, S. 512) – give a good impression of a versatile pedagogically rich treatment of conic sections respecting the different approaches. Accordingly, Treutlein connects plane geometry with spatial geometry by geometrical transformations as reflections, creates references to applications and uses folding and models for the education of internal intuition and view (*Anschauung*) (Weiss, 2016). Also, descriptive geometry, that was only taught at *Realgymnasium* and *Oberrealschule* (secondary schools with a focus on science) can be found in the appendix of the third volume of the geometry text book of Henrici and Treutlein. Here we find (without exercises) an introduction in different projection methods and hence an integration of this approach.

The dawn of conic sections

From the beginning of the twentieth century until to the New Math in the Seventies, one can find planimetric, stereometric, analytical, affine, perspective, projective up to group theoretical conceptions of conic sections, mostly close to the treatise of Walter Lietzmann's *Elementare Kegelschnittlehre* (Lietzmann, 1949). Until the end of the sixties, one can speak of a bloom of conic sections. The New Math brought conic sections in relation with differential and integral calculus as well as considerations with set-theory and geometrical transformations. Spherical geometry served as contextualization of contents and methods that were acquired in the theory of conic sections (Athen et al., 1967). Not well-known are the international endeavors in the New Math reform (De Bock & Zwaneveld, 2017) to strengthen the application side of New Math. Also, in general secondary schools of the GDR the basics of descriptive geometry where taken up in grade 7 and 8 when the school subject Technical Drawing was introduced. With the reform of the upper school in 1975 and unified examination requirements the conic sections were more and more reduced to linear structures in analytic geometry and in the

analysis to the investigation of function graphs of parabolas (Schupp, 1988) and have nowadays completely disappeared as teaching subject.

Would it not be time for a revival of conic sections at secondary school since they captivate us by their versality, systematic, and exemplarity?

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