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TEXTBOOK ANALYSIS IN UNIVERSITY TEACHER EDUCATION

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Textbooks as mirrors for modern educational reforms

In German middle and secondary schools, mathematical textbooks are extensively used in lesson preparation, in the classroom as well as for students' homework. Lesson planning for mathematics was and is generally done by mathematics teachers at school based on the mathematics textbook series used in that particular school. Also in preparation of the exercises for homework, tests and tasks for individual learning, mathematics textbooks still serve as an essential basis (Rezat 2009). An internal commission at school, consisting of all mathematics teachers, a parent and a student representing their respective groups, makes the selection of the mathematics schoolbook. In the last two decades, publishers and authors of textbooks had to adapt textbooks to competency models, reduced curricula and output orientation. The mathematics teachers at each school in particular were asked to prepare concretised school curricula based on very general educational standards and competency models. That way since the so-called Pisa shock, the German education system has undergone subtly comprehensive restructuring, the concept of "Bildung" (usually translated as "education") being replaced by the notion of "Ausbildung" (training) accompanied by a gradual economisation of the educational system during the last decade. All this had and has implications on language, approaches to problems as well as on the knowledge relevant to action, prognosis and orientation of our student mathematics teachers. Mathematics education is a reflective science. An important goal of university courses in this area should support the discourse about educational reforms and related changes to educational values. The analysis of the last editions of various modern mathematics textbooks is an excellent way to understand the implications of these reforms on general education (Allgemeinbildung) and expertise, it supports a comparative view on the everyday world and allows to disturb widespread routines.

Mathematics Textbooks as the continuous path connecting the former pupil's life with the life to come as a teacher

Taking into consideration the long German tradition of textbook development and the use of schoolbooks in mathematics classes (cf. Otte 1981), it is even more astonishing that there are hardly any canonical subjects in mathematics teacher education at university related to mathematics textbook analysis. A major problem of teacher training is the often cited double discontinuity:

> The young university student finds himself, at the outset, confronted with problems, which do not remember, in any particular, the things with which he had been concerned at school. […] When, after finishing his course of study, he becomes a teacher […] he will be scarcely able, unaided, to discern any connection between this task and his university mathematics (Klein 2016, p. 1).

As an approach to deal with the double discontinuity we look at mathematics textbooks as the continuous path connecting the former pupil's life with the life to come as a teacher. Because of the long tradition of some very wide used textbook series like "Die Elemente der Mathematik" and "Lambacher Schweizer" students can work on different editions of one textbook they were learning with as a pupils in school and they are likely to work with as a teacher. The study of "familiar"

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textbooks from the perspective of a teacher or author enables a direct approach to discontinuity and makes available various tools to support the maturation from student to teacher.

Mathematical Textbooks as historical artefacts

Mathematical textbook series are also historical sources to study the history of education. Of the textbook series "Elemente der Mathematik", one can easily find hard cover versions of different editions tracing back to the middle of the 19th century (Reidt 1868) as well a digitalised version of Reidt's exercise book (Reidt 1884). A comparative analysis of the different editions related to important periods and reforms in mathematics education, like the "New Geometry (Neue Geometrie)", "reform pedagogy (Reformpädagogik)", "transformation geometry (Abbildungsgeometrie)" "Algebraic Analysis (Algebraische Analysis)", "functional thinking (Erziehung zum Funktionalen Denken)", "the introduction of differential and integral calculus", "Mathematical applications", "New Maths and set theory", "Modeling and Realistic Maths" and "Output and Compentcy orientation in mathematics". It is also quite instructive, from a cultural-historical perspective, to compare the design of the exercises and applications from different periods. Moreover, comparative studies between reforms and traditions in different national educational systems as done by Gispert and Schubring (Gispert & Schubring 2011) can inspire the search for traces and implementation of described reforms in the textbooks of the studied countries.

Textbook Analysis as preparation for teacher practice

Teacher education for German Gymnasia consists of two parts in the federal state Rhineland Palatinate: a university study with a Bachelor's and a Master's degree (5 years) and a teacher training as interns (18 month). University teacher education in Germany has to deal with two basic principles at once, which partly exclude each other: On the one hand, courses in mathematics and mathematics education introduce students to scientific disciplines according to the concept of unity of research and teaching, whereby research should aim at insight and not at profit and usefulness. On the other hand, instantly after obtaining the university degree teachers have to teach at own responsibility because of cuttings in education and the reduction of teacher training from 2 years to 1.5 years. No wonder students expect to be trained in practical matters as lesson planning and primarily consider everything related to teacher training as useful.

Textbook analysis gives the possibility to combine the training of practical skills with the study of history of education, concept development and research design. Nevertheless, from our experiences related to reflection in lesson planning as well as on mathematical concept development only very few prerequisites can be assumed. Assessments of the capabilities of our students show that they master the reproduction of information and texts very well. They work hard on the perfection of presentational skills. Their strengths also include the use of modern media to access information and pattern recognition skills. Their weaknesses lie in their conceptual understanding.

Volker Ladenthin’s description of contemporary students’ problems confirms our experience:

> Students are barely able to use abstractions. One has to speak in examples - and they will be happy to discuss on the level of examples. However, generalization and transfer of expertise hardly succeed. To transmit the statements of ancient authors (Aristotle) in contemporary parlance fails less due to fragmentary historical knowledge than to the lack of transferability. Textual analysis is done very vaguely and always very generally, (“Comenius says that school is good for the people”). Syntheses are created additively and is by no means nuanced. Judgments are linear (not multi-perspective) (Ladenthin, 2014, p. 17).

The analysis of the design of a mathematics textbook, its presentation of basic information, classification of exercises, implicit or explicit concept development, the choice of examples to work exemplary as well as the choice of contexts and applications support conceptual understanding of mathematical notions. The latter can be guided and adapted to different prerequisites by the choice of material, questions for reflection, tools for structuring, and the provision of solution schemes.

Mathematical textbooks as a tool to change perspectives from student to author

Textbook analysis is also an excellent way to change perspectives from that of a student (solving tasks) to that of a teacher (lesson planning with tasks) to that of an author: what is the conceptual understanding of the notion? Which tasks are in the zone of proximal development of a possible student?

In our experience, this change of perspective does not happen automatically. Quite often students do the didactical analysis of a mathematical concept development not from the perspective of the teacher, but from the pupil's perspective. A typical approach of our students to lesson planning is the search for an "activating" introductional problem in textbooks or the Internet. "Activating" means here "making the pupils active in a psychological sense" and is related to methodological criteria, such as the form of cooperation or the integration of material tools. From a pedagogical point of view the students act from a teacher's perspective, since they try to organize and arrange activities and cooperation of pupils in the classroom. Thereby the search for methodological suitable "activating" tasks and exercises is often accompanied by the search for their solutions, also in textbooks or the Internet. The level of difficulty and time necessary for the solutions of the activating exercises are often assessed intuitively and not on the base of a self-conducted detailed solution. The work with mathematical textbooks provides possibilities to make the different perspectives explicit and to differentiate them from each through different tasks and activities with the textbook. Pupil's views are taken up by solving tasks and exercises and approaches. The perspective of a teacher comes with the comparison of different solutions, their systematization and their study as part of the concept development of the involved mathematical notion. The perspective of an author is taken when comparing different textbooks and analysing the roles of the different examples and tasks.

Explicit demarcation of the different perspectives and assistance in the form of reflection questions help students to assume the role of the teacher not only in the context of classroom management but also as experts in school mathematics and its suitable presentation as well as to feel responsible for the latter.

Development of a Concept of a seminar on textbook analysis

In the following, we present the concept of a seminar on textbook analysis, which has been held and developed in action research over seven years in 17 different groups. It is part of master’s degree studies at the university Mainz in Germany. The aim of the seminar is to get acquainted with different mathematics textbook designs with regard to implicit or explicit introduction of mathematical concepts, different contextualisation, systematisation and formalisation of the contextualised concepts, differentiation in exercise tasks, as well as the comparative analyses on specific topics. The sessions of the seminar are held by one or two students in form of workshops.

During the semester (6 months) before the seminar, there is a lecture course about chosen mathematical concepts from the curriculum of secondary school maths. In this lecture course research methods of mathematical education as well as "Stoffdidaktik" and task-related aspects of mathematical concept development are presented and discussed exemplary. The seminar deepens the topics of the lecture course, so the subjects of the 14 sessions are close to those of the lectures. The present choice of topics and examples in the lecture course is one of the results of an action research in the seminars and an answer to the question which mathematical concepts of secondary school mathematics are suitable for a comparative analysis of textbooks. In the first seminars 7 years ago we primarily tried to find subjects and topics combing goals of mathematics teacher training like lesson planning with educational objectives of the scientific discipline mathematics education. Therefore the first seminars studied a wide range of topics concerning analysis, linear algebra/analytic geometry as well as stochastic and concentrated on suitable topics for analysis and variation mathematical problems and tasks from actual mathematics schoolbooks. In the following reflection of the seminars it became obvious, that most of the students had problems to analyse even

canonical exercises, prerequisites, learning goals and methodology a fortiori to vary them. A first step was to limit ourselves to the analysis of tasks and to try variation of tasks only for chosen suitable examples.

Another result of the reflections was the study not only of exercises and practice but also other parts of concept development of a mathematical notion in mathematics schoolbooks.

The typical design of actual German mathematics textbook is: a) repetition b) introductive and propaedeutic problems and tasks, c) unification, systematisation, formalisation of the introductive problems, d) standard solutions, e) presentation of basic knowledge f) exercises, d) excursions.

Therefore, in the changed concept students could choose which part of concept development they wanted to explore. Most of them decided to study introductory and propaedeutic problems. As a result of the analysis and discussions of the student's essays we decided also to limit the topics to analysis and to discuss only basic notions which were preliminary discussed in detail during the lecture course: real number, function, limit of series, differential and integral. The current concept of the seminar students compares - for a given mathematical notion, like limit - its presentation in three different modern textbooks. This analysis is guided by general questions about introductory problems (implicit or explicit concept development, inductive or deductive approach), tasks and solution schemes, tools for generalisation and unification, categories and differentiation of exercises, excursions. The analyses of this common subject are discussed during the seminar and are part of an essay made by every student. In addition, in every session a student presents her or his textbook analysis related to one of the mentioned mathematical concepts and on its basis a plan of its concept exposition for the classroom. Additional materials are older textbooks, didactic articles about the mathematical notion and the material from the lecture course. The comparative analysis of the concept development in the schoolbooks with the alternative developed by the student constitutes the second part of the essay. The earlier described student's, teacher's and author's points of views are structuring tools to support the change of perspective from pupil to author.

The work from a pupil's perspective, i.e. detailed solution schemes of exercises to the concept to be discussed in the seminar are done in advance of the sessions as homework of all students.

Conclusions

Working with a textbook also provides tools to deal with different levels of awareness (Mason 1998). It takes a longer time and accompanying guidance until the students themselves ask questions concerning not only the solution of a task but also about the educational value, the development of meaning and about the existence of mathematical objects. The continuous reflection on the handling of textbooks in the frame of action research supported this maturing process. The developed criteria and categories for textbook analysis take into account the expertise and skills of the students and support the transition between using textbooks as a student and using the textbook as a teaching tool.

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