Back to the future – a journey from current education reforms to reformations in the past

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Abstract

Learning from history does not automatically mean that history prevents us from repeating mistakes. We cannot see what happens in the future, even with the most profound knowledge of the past. Although it is not possible to make such causal connections, the study of structural components, which recur and make up patterns, can certainly contribute to sharpening political judgement. How can the teaching of the history of mathematics education then help to support an understanding of possible courses of individual actions without indoctrination through the political or even ideologically influenced production of time references? The paper presents the concept of a lecture course in mathematics education, held at the University of Mainz. We take as a point of departure the everyday experience of our prospective mathematics teacher with various current education reforms and present seemingly similar processes during former reforms. Here we limit ourselves to reforms during the 19th and 20th century.

The history of mathematics education as a scientific discipline

The history of mathematics education as a scientific discipline with specialised conferences and journals emerged mainly in this millennium. First becoming internationally visible at ICME 10 in 2004, in Copenhagen, as a Topic Study Group and by the founding the international *Journal for the History of Mathematics Education* in 2007, it attracts new interest today as a subject for international meetings, e.g. at the ICME, HPM, CERME and ESU conferences. Since 2009, the International Conference on the History of Mathematics Education (ICHME) has regularly taken place every two years in different countries: in Iceland (2009), Portugal (2011), Sweden (2013), Italy (2015), The Netherlands (2017), France (2019), and Germany (2021).

The study of the proceedings of these conferences as well as the *Handbook on History of Mathematics education* (Karp & Schubring, 2014) and the two publications following ICME-13 (Furinghetti & Karp, 2016; 2018) offer insights into the development of this discipline over recent decades. They provide overviews and numerous references for further reading on the history and methodology of the field, mathematics education in different epochs and in different regions, the history of teaching mathematical subjects in school, the history of international cooperation in mathematics education, the history of tools and technologies in mathematics education as well as the history of mathematics teacher education. But doesn't the history of teaching and learning mathematics belong to the history of mathematics?

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Traditionally it does not, or, if it does, it does so in a somewhat limited way. Bruno Belhoste (1998), criticising the lack of consideration of teaching, drew attention to the social and intellectual space in which the production of mathematics occurs. Regarding what he called the socialisation of mathematical knowledge within communities of specialists and communities of users, teaching was understood by Belhoste as a special modality of the socialisation of knowledge in which the recipient finds himself in the situation of learning (Schubring, 2019, p. V).

Belhoste recommended developing the field along three (here simplified) lines: institutional history, the development of mathematics and its elementarisation through teaching, and the corresponding impact of teaching activities upon the development and diffusion of mathematical practices. Building on Belhoste's research program and its critical examination, Gert Schubring suggests an interdisciplinary socio-cultural approach based on Niklas Luhmann's sociological systems theory of science. From this perspective, communication constitutes the basic activity of science. The volume "Interfaces between Mathematical Practices and Mathematical Education" (Schubring, 2019) of the series *International Studies in the History of Mathematics and its Teaching* offers broad perspectives on all three mentioned lines of research in the history of mathematics education. Here, Schubring (2019, p. 123) gives a detailed elaboration of the socio-cultural approach using several examples from the history of mathematics.

Another approach to the history of mathematics education that was particularly widespread in Germany is that of *Stoffdidaktik* (Systematics in school mathematics). Historical teaching texts and teaching materials are analysed with the aim of making them usable to improve current teaching. Numerous works can be found here, particularly in the 1970s, in the context of the teaching reform movement *New Math* and its critical attempt to define and to locate mathematics education as a scientific discipline (Schmidt, 1981).

The historical-cultural school offers a very conceptual view of the relationship between the history of mathematics education and questions about today's teaching. Here teaching and learning are considered part of the same process, connected by interrelated processes of signifying and meaning-making (Radford & al., 2011, p. 149). In this process, words, signs, every mediating tool are understood as the products of the social historical praxis. From this point of view, the process of learning conceptually integrates its history not only on an individual but also a socio-structural level (Seeger, 2011).

The development of different approaches to and varying schools of thought in the history of mathematics education as a scientific discipline can be seen as an expression of the current general trend towards ever greater differentiation of the sciences and the emergence of new academic reference sciences. In the academic discipline *history of mathematics*, social and cultural contexts, technical and economic conditions and developments come more and more into focus. The history of education and institutions, the history and philosophy of science,

political science, educational science, linguistics and psychology provide new methods for the systematic study of increasingly complex relationships. On the other hand, globalisation and digitalisation have of course also an impacted on the development of science, especially one as recent as the history of mathematics education. Ever-extending access to additional sources for historical research, as well as the increasing importance of quantitative studies and descriptions, promote development models based on general cross-national comparable factors.

Research questions from this perspective often try to present developments in the teaching and learning of mathematics in the broadest possible context. This is especially true for contributions that are aimed at an international audience and describe comparable, cross-cultural developments in the history of mathematics and its teaching.

Preliminary considerations concerning the development of mathematics education university lecture courses

In discussing the development of suitable forms of university teaching about the history of mathematics education, we include local social conditions. Therefore, we take a different perspective on developments in the history of mathematics education, one based on action research and using social historical approaches. Nevertheless, the systematics and categorisations that emerged from the global and conceptual perspectives on the history of mathematics education are also of fundamental importance for our course of action.

The use of action research methods for the development of lecture courses is also motivated by the significance of historical social and cultural conditions and the teaching traditions of practicing mathematics educators. In this respect, the situation in the history of mathematics education differs from that of the history of mathematics. Research in the history of mathematics rarely explicitly refers to modern developments in mathematics. For the practicing mathematician, the historical genesis of a mathematical problem is seldom relevant for its solution. For most mathematicians, the history of mathematics, mathematical traditions, the biographies of famous mathematicians, their institutional anchoring, and the socio-cultural context of their work have no relation to current developments in mathematics.

This is fundamentally different from the situation in the history of mathematics education. Mathematical subjects in school have a long tradition. They are the result of negotiation and implementation by local actors. Among other things, education policy-makers, textbook authors, teachers, as well as the social conditions of the students have an important impact on the shaping of the school system. Almost all current subjects in the German curriculum of mathematics have been taught for over 100 years. Some forms of schooling and examinations have changed in the different German Federal States, but many regulations have a long tradition. The potential of current teaching subjects and

learning and teaching methods is therefore also encoded in local historical conditions and existing teaching and learning traditions. In the German education system, teachers have a high degree of personal responsibility for teaching and have the legally guaranteed freedom of methods (Gasser, 1982). Schools, as institutions, also have a high degree of independence and responsibility. A large part of the actors in local education policy completed a teacher's degree. In addition, most authors of standard school textbooks are teachers. Also, instructions during professional development, an 18-month traineeship after the academic education, are given by qualified teachers in service.

Therefore, in our approach we use subjects in the history of mathematics education that have an explicit reference to current developments in mathematics education. Our intended readers are mathematics educators with historical interests. However, we also hope that historians will find interesting questions for investigation in our approach. Of course, special local conditions and regional historical sources play a role in this approach. For us, these are the German mathematics textbooks, curricula, and other documents and secondary sources related to the school system in Rheinland Pfalz [Rhineland-Palatinate] and other Federal German States. However, our approaches have an exemplary character and are also applicable to other universities in German Federal States as well as to other countries in which the practical training of mathematics teachers is preceded by an academic education in mathematics and educational science. One of our goals is to raise awareness in our students about their future responsibilities in the development of our educational system through a cultural-historical perspective on ongoing developments in mathematics education. In particular, we are interested in historical educational reforms.

In this paper, we present concepts of and experiences with university lecture courses for prospective Gymnasium (pre-university education) mathematics teacher students, which were held at the Johannes Gutenberg University of Mainz and the University of Bonn. Another goal of the lecture course is to make prospective mathematics teachers familiar with the tradition of their profession, the development of the German Gymnasium as an institution, and changes in the school mathematics curriculum for the Gymnasium – particularly related to geometry and high school analysis. The period we studied covers the last 200 years. Because of the strong interest of many students in methodological problems, we also traced back e-learning to its roots as programmed learning.

History of mathematics as an academic subject in German universities

Thinking about the establishment of the history of mathematics education as a scientific discipline in German university teacher training, it is first worth paying attention to the academic tradition of the history of mathematics in German universities.

> The teaching and research area History of Mathematics has in Germany a long and worldwide recognized tradition. Leading mathematicians had always been interested in the history of their subject. Some of them, like Bernhard Riemann, Georg Cantor, Felix Klein, Max Dehn and Otto Toeplitz, have written important historical works themselves. With Moritz Cantor, the first chair for history of mathematics was established and Cantor's four-volume History of Mathematics was the standard reference in this area for a long time. (Purkert & Scholz, 2009, p. 215, transl. by the author)[1]

In the *German Democratic Republic* (GDR), courses on history, philosophical aspects, and the logical foundations of mathematics were obligatory inclusions in the curriculum for all students mathematics teachers (Schreiber, 1996). There were research departments for the history of mathematics in both halves of Germany, in Hamburg, Munich, Leipzig, and Berlin, as well as in various related teaching positions at universities. During recent decades, most of these positions were lost because of – among other things – staff cutbacks or reallocation into mathematics. In Germany, there is now a single professorship in *The History of Mathematics* at a mathematical institute, namely in Mainz. There are also a few historians holding teaching or research positions or teaching assignments in mathematics education at universities (e.g. Wuppertal, Koblenz, Hildesheim, Jena) as well as mathematics educators and mathematicians who include the history of mathematics in their courses (e.g. Bonn, Braunschweig, Darmstadt, Essen, Gießen, Siegen, Halle, Hildesheim, Leipzig, Mainz, Wuppertal). However, since the history of mathematics is no longer part of compulsory university mathematics teacher training, this is rarely a systematic study of the history of mathematics. However, the numerous materials based on the rich university teaching tradition of the history of mathematics allow a multifaceted integration of history of mathematics into university teacher training (Wußing, 2008; Struik, 2013). The diverse and rich German language materials on the history of mathematics, which are the result of this long tradition, also form a good basis for the academic teaching of the history of mathematics education.

[1] Das Lehr- und Forschungsgebiet Geschichte der Mathematik hat in Deutschland eine lange und weltweit anerkannte Tradition. Führende Mathematiker haben sich stets für die Geschichte ihres Faches interessiert und einige, wie Bernhard Riemann, Georg Cantor, Felix Klein, Max Dehn und Otto Toeplitz, haben selbst bedeutende historische Arbeiten verfasst. In Deutschland gab es mit Moritz Cantor den ersten Professor für Geschichte der Mathematik, und Cantors vierbändige Mathematik-geschichte war lange Zeit das Standardwerk auf diesem Gebiet.

Traditions of academic mathematics teacher training at German universities

After a short overview of the teaching traditions of the history of mathematics in German universities, we also take a look at other reference sciences of the history of mathematics education and their tradition of academic teaching. Teacher training for the Gymnasium in Germany has undergone several reforms since the end of the 18th century, the main aim of which was to academise the teaching profession. Mathematics teacher training at university was therefore oriented towards science education. This can be seen, for example, in the introduction of the "examen pro facultate docendi" in 1810 [professional state examination] and the Abitur examination in 1812 in Prussia. Corresponding to the predominance of the ancient-philological humanistic gymnasium, philological teacher training initially dominated these exams (Hamann, 1993). Due to technical industrialisation during the 19th century and the emergence of polytechnics, and with them preparatory *Oberrealschulen* and *Realgymnasium*, mathematics and the natural sciences became more important in secondary school teacher education. With new Prussian examination regulations in 1831 and 1866, the scientifically educated scholar became – and is still – the role model for *Gymnasiale Lehramtsbildung* (high school teacher training) at German universities (Berthold, 1993; Apel, 1985; Jessmann, 1999). Teacher training for high schools in the former GDR had a larger pedagogical foundation, but after reunification, regulations in the new German Federal States were aligned. It was only with the conversion of the state examination courses into the bachelor-master system at the beginning of this millennium that the educational sciences and subject didactics in the academic teacher training courses gained more space and attention.

In high school mathematics teacher training for the *Gymnasium*, scientific education remains in the foreground and lecture courses in science are largely the same as for the science students. The studies of mathematics, mathematics didactics, and educational sciences are traditionally isolated from one another.

This is often seen as the cause of a lack of interest in many teacher students with the "non-practical" study of mathematics. On the one hand, this is due to the fact that teacher training is not seen as a field for mentoring, academic support, and integration of emerging researchers in mathematics. On the other hand, in spite of the principle of the "indivisibility of teaching and research," there is a widespread view that the teaching of mathematics has no influence on mathematical practices and their development.

> The contents of teaching are seen as a certain kind of projection of academic mathematics, as a certain sedimentation. Therefore, the relation between the development of mathematical practices and the teaching of

> mathematics is often conceived of as unilateral, without an impact of teaching upon research (Schubring, 2019, p. 123).

Thus, dealing with the historical role of teaching and communicating mathematics with respect to historical advances in mathematics could help to strengthen this interest, due to the gap between "production" and "reproduction" (Schubring, 2001), between academic studies of "higher" mathematics and the teaching of "elementary" mathematics in school.

Local conditions and general requirements at Mainz University

In our view, the ideal form to make students familiar with the history of mathematics education would be within the framework of the master's degree, in form of a specialisation as is common for the history of mathematics. Such a course could contain a general overview about the history of teaching mathematics and teaching tools, about the history of the institutionalisation of mathematics as a school subject; it could exemplarily demonstrate the methodology of the field. Nevertheless, due to organisational requirements in Mainz as well as in Bonn University, the history of mathematics education at the current time can only be taught systematically in mathematics education modules. This implies that historical topics must relate to basic topics of mathematics education.

The lecture course is placed at the beginning of the bachelor's degree, therefore we cannot assume knowledge of the history of mathematics nor of higher mathematics. We developed the concept of a lecture course including history of mathematics education in Germany by means of the methodology of action research. The progress of the course was discussed regularly in a joint seminar with other mathematics educators, who had parts of the course in the form of excursions integrated in their lectures. Since there are curricular standards in university teaching of mathematics education that have to be covered, the lecture series had to be taught to students in their first or second semester only as an lecture course without exercise classes, consisting of 13 lectures. The work of the students was evaluated by a written examination (2 hours).

Concept of the course

The lecture course has five subject areas. We provide the recommended reading material for the students, but only partially cite the literature used in the lectures, as the latter would be too numerous. The first two lectures deal with the prejudice of apolitical education in mathematics classes and concepts about the absolute and objective nature of mathematical truths. The prevalence of the latter concepts of mathematics among prospective mathematics teachers and mathematicians is clear in several studies (Stroop, 1998; Heintz, 2000).

In the first lecture we discuss historical examples, which show how materialistic (Engels, 1878), religious (Kessler, 1999; Szabo, 1992; Teichmann, 2013), racist (Hamel, 1933; Mehrtens, 1987; Segal, 2014), sexist (Möbius, 1904), and biological (Branford et al., 1913) ideologies influence the way in which mathematics is taught. Examples from historical mathematics textbooks show how mathematical tasks can be misused as a means of military service and disciplining (Schappacher, 2003).

The second lecture addresses the question of whether mathematics education can support populism. The problem is studied from a conceptual mathematical-philosophical perspective and uses Paul Ernest's interest groups (Ernest, 1991). This classification of different perspectives on the nature of mathematics offers the possibility of classifying historic figures' conceptions about mathematics, as well as to tracing them in contemporary educational programs.

The second set of topics is devoted to learning theories in mathematics education. Here too, a cultural historical perspective is taken. We discuss exemplary historical contextualised pedagogical and psychological theories (J. Piaget, L.S. Vygotsky, P. van Hiele, A.N. Leontiev, V.V. Davydov, P.J. Galperin), which are important for mathematics education and represent cognitivist, constructivist, and socio-constructivist perspectives. In particular, we study the cultural-historical school, allowing us to later on adopt a reflected cultural-historical perspective (Jantzen, 2012).

The third set of topics is devoted to concept development. The mathematical concept of symmetry is well suited to demonstrate the differences between current and selected historical approaches. Currently, symmetry in textbooks occurs almost exclusively as a property of mathematical objects. The learning objectives then consist of recognising and naming the properties of symmetrical objects and in distinguishing and ordering mathematical objects with regard to the presence of symmetries. A look at textbooks at the end of the 19th century beginning of the 20th century shows that symmetry also has another function – as problem-solving method, as a tool. Even in the 1960s, in the context of the New Math reform, symmetries as mathematical structures and as the basis of transformation geometry were of great importance. Our historical sources for the study of historical approaches are textbooks of the periods mentioned and the popular scientific historical account by Ian Stewart (Stewart, 2008). In the investigation of the concept of symmetry in the context of school mathematics, cultural-historical considerations of historical drawing and measuring instruments are also included (van Randenborgh, 2015; Vollrath 2013).

The development of *programmed teaching* in Germany as one nucleus of today's e-learning is a further subject of the lecture course. Here we study connections between the New Math movement and the development of programmed learning on the grounds of behavioural learning theories (Correll, 1965; Correll, 1969; Damerow, 1977).

The last set of topics is devoted to the role of conic sections in curricula and mathematics textbooks for German gymnasium of the 19th and 20th century. In

this context, we also study historical developments of the profession *Gymnasial-lehrer* in Germany during this period.

In the lectures and the reading texts, we regard the historical reforms as changes in institutional structures, as well as ideological movements. Our historical examples are the institutional reforms of the German secondary school system during the 19th century, the reform of the teaching of geometry in school during the 19th century, and the Meraner Reform and the New Math reform as examples of curricular reforms. Due to time constraints, we do not look at these reforms as reactions or even countermovements to previous reforms.

By choosing perspectives on historical reforms and moderating the corresponding discussions, attention can be drawn to features of reforms that we think are worth considering in terms of current reforms. These include, for instance, theoretical foundations, organisational preparations, coordination with regard to content and methodological aspects, and transparency of responsibilities. In particular, we focused on the aspects of recent and historical reforms mentioned below.

The subsequent recent reforms can be seen as institutional reforms of the school system: The abolition of the *orientation level*[2] and pre-school education, the reduction of upper secondary classes by one year, the restructuring of secondary schools, the shift from special schools for specific disabilities to integrated/ inclusive forms of schooling, and the shift from the three-tier school system towards a comprehensive school. Every former reform of the education system was related to just one type of school and maybe related types. They were carried out over a period of 200 years. The current reforms took place almost simultaneously over a period of just 20 years and involved all school forms at once.

Historical sources can help to relate institutional reforms to the biographies of their main actors and to political and economic contexts. It is often difficult to find historical sources with schedules, programs, and resolutions, but if you do find them, you can discover the main actors or at least their names. This is not the case with present documentations of reforms. Here, the authors hide behind huge organisations and their programs. It is hard to find out who is responsible.

The search for historical educational reforms that relate to a very general principle leads us to reforms of school geometry driven by the slogans "Neuere Geometrie" [New Geometry] or "Los von Euklid" [Euclid must go!], both in the 19th century (Kitz, 2015), the Meraner Reform and its motto of "Erziehung des funktionalen Denkens" [Education of functional thinking] (Krüger, 1999; Schubring, 2007), or the "Neue Mathematik" [New Math movement] of the 20th

[2] Grade 5 and 6 of secondary school were supposed to be an orientation period in order to decide about the type of school.

century (Correll 1965; Corell 1969; de Bock & Vanpaemel, 2015). The principles of former reforms are inspired by developments in mathematics such as projective and other (non-Euclidian) geometries, descriptive and analytic geometry in the 19th century and algebra, logic and set theory, and probability theory in the 20th century. The current reforms are triggered by the psychologisation and economisation of social processes. On the contrary, the pretext of the current reforms, the so-called "PISA shock", sounds quite similar to the "Sputnik shock" on the eve of the New Math reforms. However, the Sputnik shock occurred whereas PISA was launched (Huisken, 2005; Jahnke & Meyerhöfer, 2007; Freudenthal, 1975).

To understand more about the preferences of our students we included questions in the written examination where they can choose their examples: "Name five historical events/reforms and their temporal placement concerning the development of mathematics education, which you find particularly impressive and justify this. Give five historical facts and their temporal placement concerning the development of your profession, which you find particularly impressive and justify this". The most popular subjects of the course were conic sections in the teaching of mathematics during the last 150 years, the history of mathematics in the teaching of mathematics during the last 150 years, history of the profession "Gymnasiallehrer" and the "Gymnasium", "New Math" in the context of programmed learning, programmed learning and e-learning.

Reforms as a recurring theme and reference item

The various topics of our lecture course were linked by referring to historical reforms. Why is it important and even necessary now to deal with the history of mathematics education? The past two decades have been characterised by a large number of educational reforms that affect all areas of the German education system: the aforementioned institutional reforms of the primary and secondary school system, several curriculum changes, as well as the change to output and competence orientation, the introduction of educational standards, the introduction of central tests, digitalisation, and inclusion as new basic principles.

Some of the reforms have already been withdrawn. There is agreement among institutional stakeholders that mathematical education is experiencing a deeper crisis. In its statement "On the current discussion on the quality of mathematics teaching" from April 20, 2017, the German Mathematicians' Association (DMV), the Society for Didactics of Mathematics (GDM) and the German Association Funding of Mathematical and Natural Science Teaching (MNU), describe the crisis of mathematics teaching as follows[3]:

[3] https://madipedia.de/images/e/e3/Stellungnahme-DMVGDMMNU-2017.pdf

> For more than a decade at German universities one notices the alarming finding that a large number of students lack basic mathematical knowledge and skills as well as conceptual understanding of mathematical content at the beginning of their university studies (transl. by the author).

The crisis is not seen in the context to the failure of some of these reforms, nor due to their short-term nature or political activism. The lack of prerequisites for an apprenticeship or study has recently been noticed and criticised by representatives of the economy, especially medium-sized businesses.

What is precisely going on in these developments? Can the study of history of mathematics and mathematics education support an analytical and a critical view on these developments?

Learning from history does not automatically mean that history prevents us from repeating mistakes. Politicians are not supposed to be historians – historical situations never completely recur, and therefore the future cannot be predicted from even the most profound knowledge of the past. Even though it is not possible to transfer causal connections, the study of structural components, which recur and make up these patterns, can certainly contribute to sharpening political judgement (Bergmann, 2002; Geiss, 2019). How can the teaching of the history of mathematics education then help to support an understanding of possible courses of individual actions without indoctrination through the political or even ideologically influenced production of time references?

Due to the current situation, we think that the study of the history of former educational reforms and their theoretical foundations help to raise awareness of ongoing changes in the German educational system, particularly its economisation.

Our students have experienced several of the cited reforms during their school time themselves. We wanted to incorporate their opinions into the concept of the lecture course on the history of mathematics education. Therefore, we take the everyday experience of our mathematics teacher students with various current education reforms as a point of departure and present seemingly similar processes during former reforms (Weiss & Kaenders; 2019). Thereby, we work backwards. We start with a description of the current situation and ask students to find differences to practices, school subjects, and events in the past, which look at a first sight very similar or have similar names. Thus, the formation of analogies does not arise through our study of historical reforms. Conversely, at first glance similar events lose their similarities through our investigations.

Final considerations

The philosophy of mathematics education seems helpful to develop a conceptual view of the history of mathematics education. It is also worthwhile to pay

attention to the attitudes of students on current ideas and to ideas about future mathematics education at schools. During the school years of our students, ongoing reforms – aimed at further standardising institutional structures under the slogan of "comparability for more social equity" – became the norm. Their desire to become a teacher arose in this education system. Therefore, today, after 15 years of an education policy based on central testing and the measurability of learning results, critical views on the economisation and digitalisation of the education system cannot be assumed. Despite this – or perhaps because of it – the study of former reforms, their aims, preparation, organisation, and realisation should have a place in university teacher education as well as in in-service teacher training.

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