HISTORICAL MATHEMATICAL MODELS IN TEACHER EDUCATION - WORKSHOP ON THE DEVELOPMENT OF QUESTIONS AND CRITICAL QUESTIONING

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We present the conception and design of a seminar in the master's programme of mathematics teacher education. The seminar starts with students' personal experiences relating to mathematical experiments, models and visualisations of mathematical objects, followed by an historical excursion around historical collections of mathematical models. On the basis of that, students undertake project work on models of drawing instruments and simple curves in historical, socio-cultural or mathematical contexts.

INTRODUCTION

The paper deals with a workshop held in the afternoon of the last day of the 7th European Summer University on History and Epistemology in Mathematics Education at Aarhus University Copenhagen. In spite of the time of the event, the workshop was well attended and met the interests of mathematics educators with various backgrounds. Our aim is to give the participants a memory of the event and to outline the concept and design of a seminar, which uses history as a tool to awaken awareness and understanding of individual development and societal change in a mathematical context. We use a comparative view on the everyday world and its past to disturb widespread routines, approaching development from an output-orientated perspective and in normative terms.

THE MAIN MOTIVATION FOR THE CONCEPT AND DESIGN OF THE SEMINAR

The aim of the presented seminar concept is to deal with normative perspectives, evaluative assessments and output-orientated categorizations of our students on education and development. Since the so-called Pisa shock, the German education system has undergone subtly comprehensive restructuring, the concept of "Bildung" (usually translated as education) being replaced by the notion of "Ausbildung" (training). At least since Klafki (1994, 2000) formal education approaches in German educational sciences are believed to have been overcome. The reality of education and educational policy are now taught differently.

Content free and subject independent language and numeracy skills are declared as basic skills, as conditions for participation in social communication (Baumert et al, 2001).

A self-realizing system of empty phrases linked to test results arrogates to itself global assessment of human development.

Historical processes allow to caricature such models of development. Reduction of complex relations and causal dependencies to input-output mechanisms can be taken in a cultural-historical context ad absurdum.

The gradual economisation of the educational system during the last decade also has implications on language, approaches to problems as well as on the knowledge relevant to action, prognosis and orientation of our student mathematics teachers.

Assessments of the capabilities of our students show that they master the reproduction of information and texts very well. They work hard on the perfection of presentational skills. Their strengths also include the use of modern media to access information and pattern recognition skills. Their weaknesses lie in their conceptual understanding.

Volker Ladenthin's description of contemporary student problems confirms our experience:

"Students are barely able to use abstractions. One has to speak in examples - and they will be happy to discuss on the level of examples. However, generalization and transfer of expertise hardly succeed. To transmit the statements of ancient authors (Aristotle) in contemporary parlance fails less due to fragmentary historical knowledge as to the lack of transferability. Textual analysis is done very vaguely and always very generally ("Comenius says that school is good for the people"). Syntheses is created additively and is by no means nuanced. Judgments are linear (not multi-perspective)" (Ladenthin, 2014, p. 17).

Based on this situation, the focus of the seminar is not on a historical outline of the use of mathematical historical models but on the development of individual questions giving rise to contextualisation. To avoid questions leading to formal reconstructions we start with our students' personal experiences with mathematical models.

It seems to be useful to divide the individual school experiences of the participants with mathematical models into experiences as pupils, and experiences as mathematicians or as teachers. An excursion to a "hands-on" museum with mathematical models, for instance, raises quite different questions for the visitor, the tutor and the organiser.

FROM LEARNING MATERIALS TO HANDS-ON EXHIBITION

Contemporary student mathematics teachers can have varied experiences with mathematical models. Reform pedagogy is an important topic in educational studies. Diesterweg, Pestalozzi and Dewey represent sense perception and activity orientation in mathematics education and their approaches are part of the curriculum in educational science. Depending on the interests of the student group and their background, one could begin with a pedagogical-philosophical orientated introduction into reform pedagogy, starting with the 19th century or with an introduction closer to mathematics - for example, with Treutlein's collection of mathematical models (Wiener, Teubner, 1912).

For both lines of discovery learning we can use the students' personal experiences with mathematical models.

	Learning materials like the *mathematical box (Mathekoffer)* or sets of platonic solids or experimental instruments are part of the inventory of most schools and are hence discussed in courses in mathematics education.	

Fig 1: examples of working materials for mathematical lessons

Some universities organize hands-on exhibitions for pupils, where student teachers are involved.

Fig 2: Some examples of hands-on activities and materials

Before the introduction of the Bachelor-Master-System and its rigid credit-course system it was easier to organize courses in mathematics education where students could develop their own mathematical experiments and try them out in a practical way in mathematics teaching.

THE SEMINAR CONCEPTION CONCERNING CONTENT AND STRUCTURE

The development and production of mathematical models were already in use in the 19th Century and early 20th Century for the training of student mathematics teachers. Nowadays, this is an episode in the history of European science. The use of historical

mathematical models and their digital images in the study, the teaching and development of mathematics allow for the relation of historical, technical, educational and information technology aspects to each other.

The workshop introduced an approach that can be used to design and produce learning materials for a seminar in university education of student mathematics teachers. There is a variety of literature with historical and pedagogical perspectives on the development of mathematical models, which constitutes the content framework for the historical research undertaken by the participants in the seminar (for instance, Bussi et al. 2010). The students actively take part in the process of material choice and organisation of the seminar. The format of the seminar supports responsible learning with initiative, learning by discovery and situational learning. However, it seems to take some time and effort to get used to it.

The participants conduct the study of historical mathematical models in the seminar by three types of contextualization. They are in particular:

1. Mathematical models as a historical artefact,

2. Historical mathematical models as a source of study and as a visualization of historical mathematical contents,

3. Mathematical models as a source of inspiration for experimentation, varying and developing new models and visualizations.

The study of the first perspective – historical mathematical models from a socio-cultural perspective – could include aspects such as:

• Teacher training before, during and after the German *Meraner Reform*,

• Intuition and perception of mathematics in the context of educational values and norms,

• Mathematical models in the context of the discussion between pure and applied mathematics,

• Historical mathematical models and patriotic education.

The second and the third perspectives focus on the mathematics behind the models and on visualisations of mathematical concepts. We introduce an additional structure through the goals of the related mathematical activities (figure 3). The concrete choice of specific models depends on the mathematical preparation of the participants in the seminar. A first attempt to organize such a seminar has been made by David Rowe, Oliver Labs and myself representing the history of mathematics, mathematics and computer science and mathematics education. The prerequisites for the students consisted of the main basic courses in pure and applied maths, courses in educational studies (pedagogy, psychology) and introductory courses in the historical and cultural roots of mathematics as well as seminars in maths education. Some of the students attended reading courses and seminars in selected topics regarding the history of mathematics.

In this first seminar, we restricted ourselves to historical models, mechanical instruments and drawing tools related to plane curves. The individual models were assigned to the themes:

• Selected static models,

• Selected kinematic models,

• Selected models closely related to school curricula.

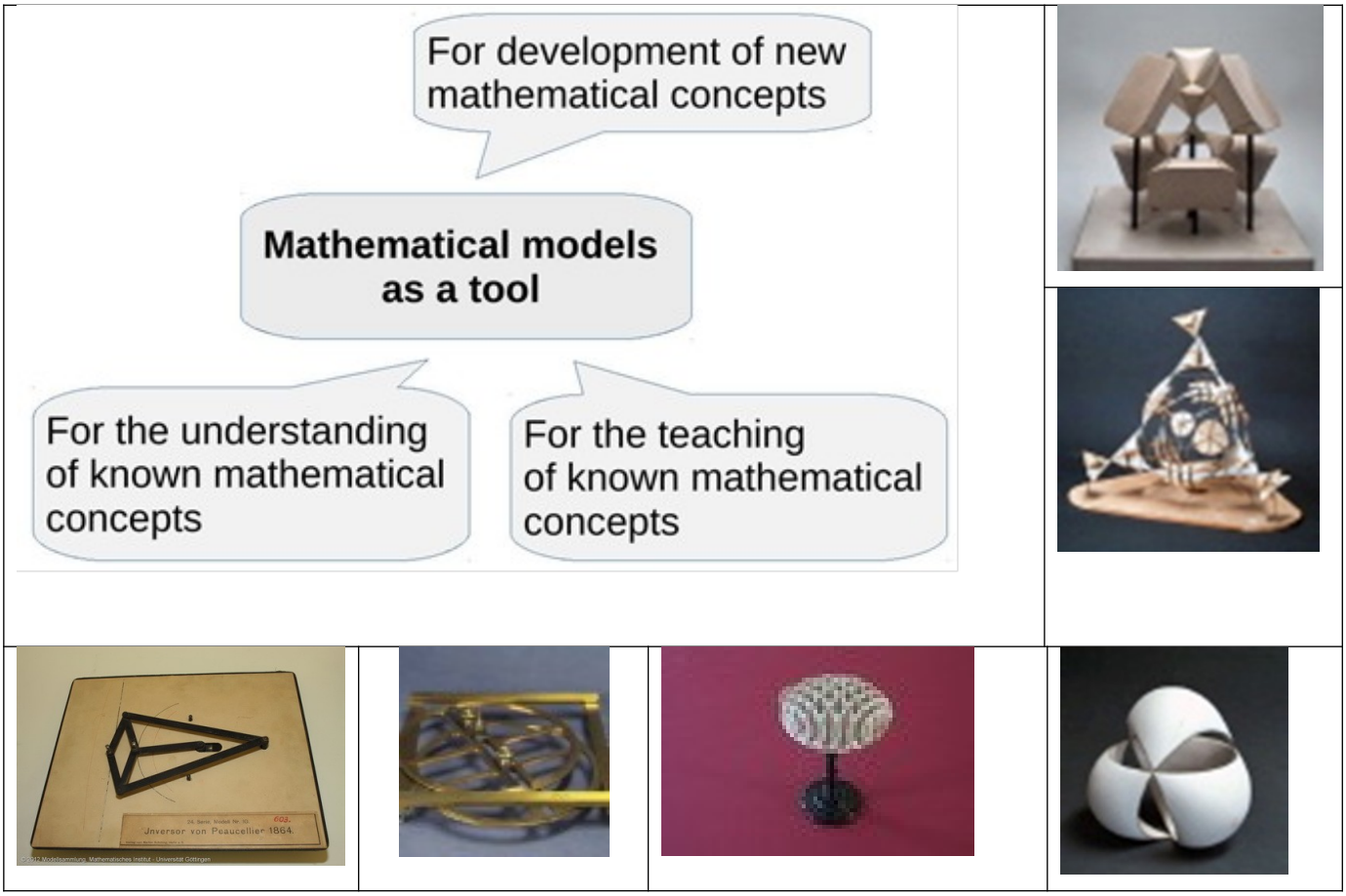


Fig 3: Perspectives on mathematical models as a tool, examples from the historical collection of mathematical models in Göttingen, Mathematical Institute.

Understanding the background of most models of the Brill and Schilling collection is a mathematically very challenging task for students. However, the visual and tactile access when working with visualizations and real models fosters this understanding. This takes place in the realm of the third contextualization while dealing with the illustrated mathematics and the development of further illustrations. The concept of the seminar allows for the possibility of discussing different themes concerning elementary mathematics, the history of mathematics, mathematical teaching methodology and computer algebra.

From the experiences with the first Seminar of this type, it seems useful to plan prior to it a reading course, making students familiar with the conceptual foundations of

using the history of mathematics as a tool and as a goal, as well as with the corresponding concrete examples of classroom practice from different countries and time periods (Jankvist, 2009. Fauvel & Maanen, 2000).

OUTLINE OF THE WORKSHOP

The workshop started with a short presentation of the concept of the seminar. As an example of a collection of historical mathematical models we chose the *Göttinger Sammlung historischer mathematischer Modelle*, which is a historical collection of historical mathematical models closely connected with the name and activities of Felix Klein (Rowe, 2013).

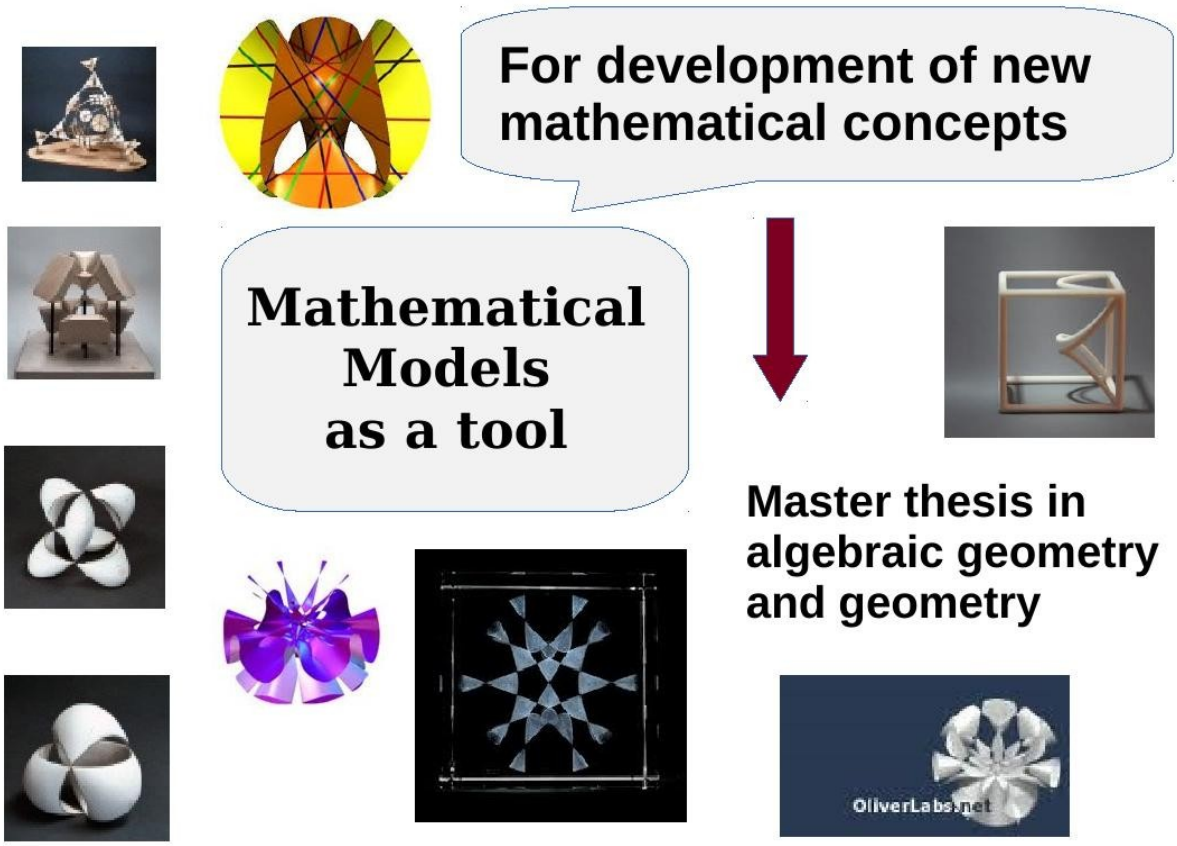


Fig 4: Some examples of historical and modern mathematical models

This collection is available digitally (http://www.uni-math.gwdg.de/modellsammlung/). Moreover, during the last year in the framework of a project at the Georg-August-University Göttingen to introduce historical collections into teaching, the collection of historical mathematical models became subject of several teaching activities in the study of mathematics as well as in didactics. The topic of mathematical models is highly suited for pushing students to formulate their own research questions. After a short introduction to the background of the participants in the workshop, we started with a short presentation of the

Göttinger historical collection of mathematical models. In order to illustrate the pedagogical method central to the seminar, the participants were invited to formulate research questions related to the models they would like to study. Most participants wanted to understand more of the mathematics visualised by the models.

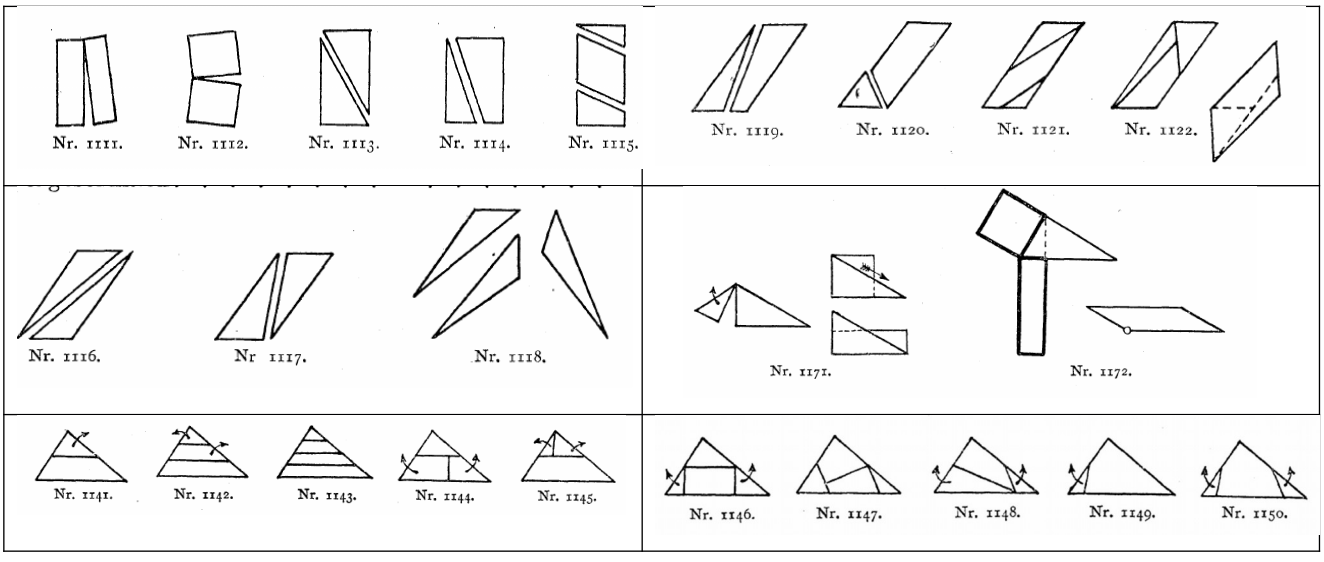
In the university seminar the situation was quite different. Most students wanted to know about the materials of the models and their durability, the form of activity (group work, individual work), the order and delivery times, the quantities in production and the prices of the models compared to other products. There were also approaches to studying the mathematical models as artworks and as a source of inspiration for artists.

The contextualisation of the models by the university students was much more social-cultural than the Copenhagen workshop.

One of the strengths of the university seminar was the team teaching and the interdisciplinary approach of the teaching staff. Depending on the type of questions raised by the students, they could consult either the historian of mathematics, or the specialist in algebraic curves and surfaces and their visualisations or those of the maths educator. For students interested in understanding more about the mathematical background of models of algebraic curves, there were possibilities to write a master's thesis on this topic and to study and develop digital images of the models and digital visualisations (figure 4).

For a workshop with participants of varying mathematical backgrounds, the historical models of algebraic surfaces were, however, too complex to illustrate different contextualisations.

The models, that were presented were not part of the Brill and Schilling collection for university education but were models from the Treutlein Catalogue, i.e. models for use in schools.



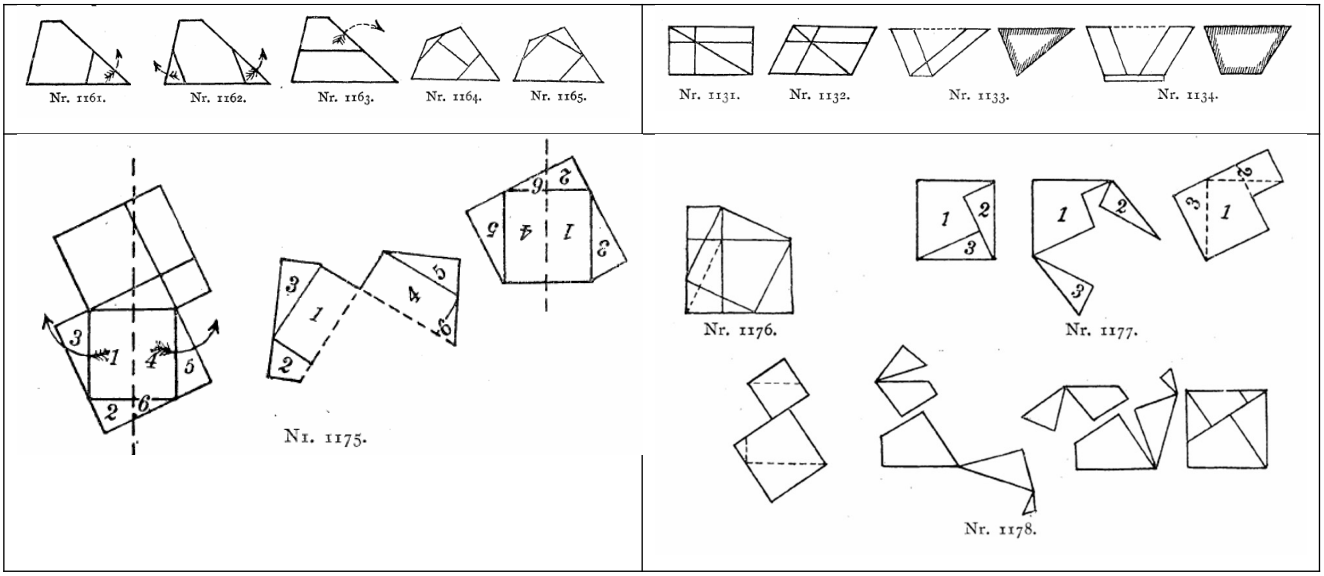


Fig 5: Samples from Treutlein‘s Catalogue (Wiener & Teubner, 1912)

The second exercise of the workshop sought to get the participants into the spirit of German school practices of the early 20th century. The participants were asked

1. To choose from the Catalogue of Treutlein’s collection a series of models from a chapter with geometrical flat models (fig. 3),
2. Following the pictures from the Catalogue to produce models from cardboard,
3. To give a short lesson plan on the basis of activities with the cardboard models,
4. To formulate corresponding tasks and exercises with the models.

The main implicit goal of this group-work was to develop a contextualisation of the chosen school model, which is related to individual activities and routines. The latter are the basis for formulating meaningful research questions on the subject of historical collections of (industrial produced) mathematical models for classroom activities.

Another reason to choose rather simple geometrical models from the collection is Treutlein’s accompanying geometry textbook (Treutlein, 1911). The historical school textbook gives the possibility of contrasting contemporary and historical mathematical activities – modern lesson planning and problem solving with given (historical tools) and the intended activities with the same tools.

The corresponding tasks during the workshop were:

- Find in Treutlein’s geometry book tasks to your crafted model.
- Compare the task with your modern versions.
- Give a formulation of a task, which could be Treutlein’s.

The first part of the workshop was rounded off with a comparison of the treatment of Pythagorean Theorem in Treutlein’s textbook with a modern mathematical school textbook which uses very similar puzzles for the proof.

The first part finished with a discussion of questions inspired by the making of models:

- In the historical context
- In the modern context.

Inspired by the use of models:

- From the perspective of students
- From the perspective of teachers
- From the perspective of authors.

The second part of the workshop dealt with the independent development of visualisations, models and experiments as teaching tools.

It started with a presentation of contemporary self-crafted experiments and visualisations for primary school and for high school students. The presented hands-on exhibition for primary school students was a project we did in Göttingen in the frame of an annual urban science week. The project was organised by mathematics educators and students of the mathematics department of university Göttingen, it started in 2003 and continues to the present. In this hands-on exhibition students developed their own experiments and worked as tutors (Fig.6). The activity initiated the development of similar school exhibitions in various primary schools through to a regular mathematical after school club (http://www.gs-friedland.de/index.php?article_id=41).

Fig 6: Experiments and models in the framework of a hands-on exhibition of mathematical experiments organized and developed for primary school students

The presented models and visualisations for high school students were results of the student work of our seminar on historical models. As we have already explained, due to the mathematical preparation of our students, we restricted ourselves to simple

mostly plane curves. The mathematical themes of the seminar and related course works were:

- Models related to problems of Apollonius
- Drawing instruments
- Cycloids
- Involute and Evolute
- Curvature
- Quadrics
- Curves in space

The students were encouraged to visualise the studied mathematical objects, to create models, to develop experiments and to support digital visualisations and experiments. They had the possibility of attending an associated computer algebra course. Some of the produced visualisations and models of studied mathematics are shown in Fig.7.

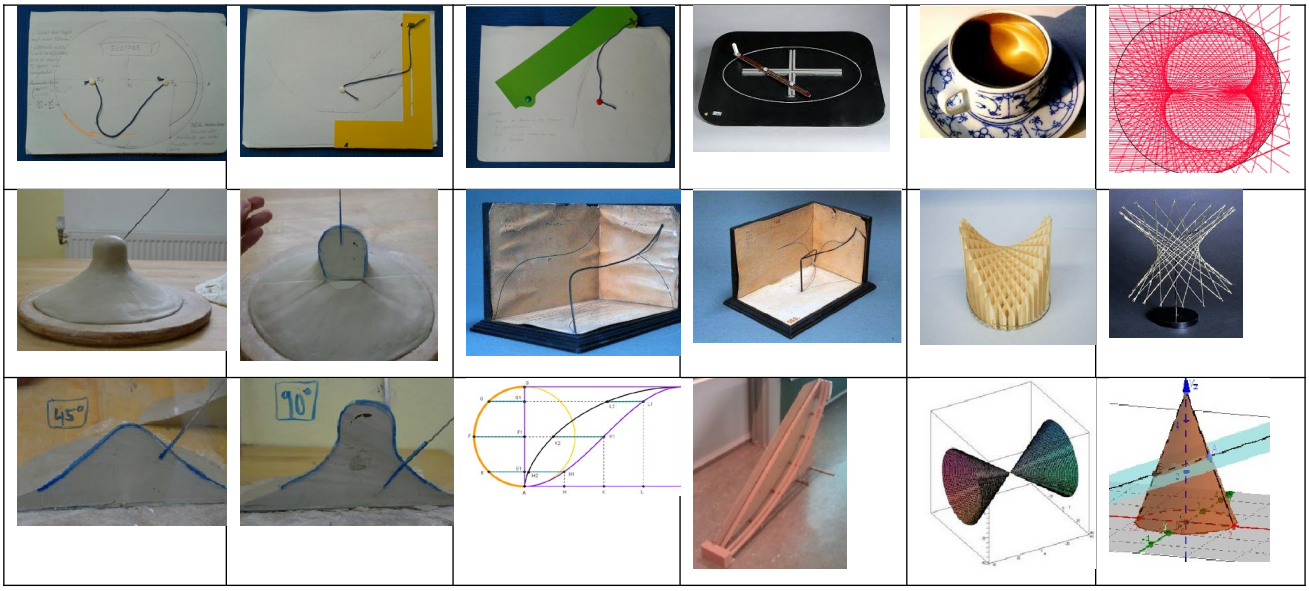


Fig 7: Models, experiments and digital visualisations produced by students in the framework of the seminar

Similar to the first part of the workshop, the aim of the introduction was to contextualise the making of models and visualisations by personal experiences and motives. The participants were asked to formulate historical, socio-cultural and mathematically motivated research questions based on their own interests and experiences.

The last task for the participants was to make their own models from paper, cardboard, coloured pencils or crayons, scissors, and binding material such as yarn or twine. For guidance, one could use the Internet, student assignments or real instruments like a pantograph.

IN RETROSPECT

After the workshop, I was asked for more material concerning the seminar, like literature, instructions for model constructions, applications for the digital experiments.

To present and illustrate the concept of an activity aimed at raising questions and not giving answers was harder than I expected. An adverse impact was the seating arrangements – one very long and small table group. Working in small groups presenting the results of group discussions with an overhead projector (as is done in student seminars) would have given the workshop more structure in form of "answers in questions".

I am especially grateful to Jan van Maanen for his enriching comments during the discussion and his inspiring enthusiasm at the production of cardboard and twin models.

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