USING HISTORICAL SCHOOL BOOK EXCERPTS FOR THE EDUCATION OF MATURE MATHEMATICS TEACHERS

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The use of historical and cultural perspectives in university mathematics education can support the development of self-esteem and maturity. It can bring together students with similar interests. We present the concept of a seminar on the analysis of mathematical textbooks and of learning contexts based on the consideration of historical excerpts. Such a seminar can become a starting point for a community of practice with the potential to develop social recognition and personal appreciation of the individual interests and talents of its members and their joint activities.

INTRODUCTION

The organizers (among whom were the authors) invited gifted student teachers in mathematics at mathematics departments nationwide to apply for a four-day spring workshop in beautiful surroundings in Bonn that was paid for by the university – including travel and entertainment expenses. Despite this exceptional opportunity only a few students answered this call. A closer investigation of this fact informed the organizers that most students, even the best mathematics student teachers, do not feel particularly *gifted*. In individual cases there might be individual reasons for this, but in general we interpret this phenomenon by the fact that these students were brought up in a school system in which they had to produce a required output in situations that are created and determined by others. The students learnt to interpret the evaluation and assessment of the learning output as a degree of their own learning success – just as how they learn in social networks to take the number of ‘likes’ on their posted output as a degree of esteem.

Mathematics student teachers for the gymnasium usually study mathematics in courses together with mathematics bachelor students, who study only mathematics (and a minor) all day. In mathematical tests and evaluations they often perform weaker. The student teachers in Germany study next to mathematics a second subject and pedagogy, where they learn completely different perspectives and methods. Instead of feeling like a jack-of-all-trades they experience their inferior results in mathematics assessments as inferiority in terms of their ‘giftedness’. Other forms of assessment as well as feedback from fellow students and professors do exist, but it seems that they are not taken as much into account as written maths examinations, as far as the self-conception of the student teachers is concerned.

Notably the *von Humboldt Bildungsideal* is built on two notions: the *autonomous individual* and the *cosmopolitan or Universalist (Weltbürger)* – that is, a universally interested person that cares about the important questions of humankind. The university should be – both for students and professors – a place for autonomous individuals to become such a *Weltbürger*. Furthermore, how do we want the students to act later as teachers, when they educate their own students? What is their understanding of *Bildung* and education at school? Student teachers, who are about to become responsible experts for *Bildung* at school, not only need to get in contact with these ideas, but should also be given opportunities to work on their own *Bildung* and personal development.

These considerations constitute in mathematics education the need for appropriate learning opportunities. Working on *Bildung* has an impact on one's whole personality and is therefore likely to last longer in professional practice. Here the history of mathematics plays an important role in mathematics teachers' education.

There are several projects that use historical themes for an autonomous study of student teachers or teachers. There are classical seminars, where students give talks on historical themes; there are websites to supply students and teachers with historical sources that can be used for teaching at school[1]. In addition, there are book projects evolving from seminars with students on historical matters (e.g. Van Maanen, 1995).

In this article, we reflect upon the set-up of a seminar around historical excerpts from schoolbooks that allows a process of value creation in a community of practice. First, we explore this idea by conceptual considerations and secondly, we illustrate the concept using Euclid's proof of the irrationality of $\sqrt{2}$ and the development of analysis.

ENLIGHTENMENT IN MATHEMATICS TEACHER EDUCATION

How can we transfer the existing learning opportunities in the above-described direction of *Bildung*?

The normal procedure in academic studies for student mathematics teachers is that they follow basic courses in pure and applied maths, courses and lectures in mathematics education, courses in educational studies (pedagogy, psychology) and study a second school subject on equal footing to mathematics. Only in some universities (e.g. Mainz) are there general courses on the historical and cultural roots of mathematics, reading courses and seminars for chosen topics in the history of mathematics. Assessments in mathematics are mostly tests on problem solving and mathematical terminology. Assessment varies in the history of mathematics and in mathematics education - we find essays, coursework, oral examination, seminar papers, presentations and homework assignments.

The feedback and assessment provided is thus much broader than in written mathematics exams. The problem is not that these alternative forms of assessment wouldn't exist. In mathematics, students hand in weekly exercises (often duos or trios) that are not only discussed qualitatively with the collaborating fellow students but also in weekly tutorials. Moreover, the students give talks in seminars. In mathematics education, in the history of mathematics, and in pedagogy courses as well as in most second school subjects, students have to write homework essays and give presentations. All this seems not to essentially affect the student's sense of self.

To sum up, with respect to *Bildung*, we encounter problems partly related to the common practices of assessment in mathematics courses:

- Student teachers appraise their own abilities according to their results in mathematical tests.
- The categories *right* and *wrong* characterize the attitude to mathematics.
- Prerequisites for teaching new content in mathematics are thought of in the form of activity-free knowledge (the student knows...).
- The bureaucratic Bachelor-Master system insinuates that education is about measurable outcomes, credits, evaluations, quality management, etc.

Do we want our future teachers to have this attitude towards their students? What are they going to teach them? Here, enlightenment ideals can still make a difference to the by now common views on education or training.

Immanuel Kant gave "An Answer to the Question: What is Enlightenment?" (Königsberg in Prussia, September 30th, 1784):

> Enlightenment is man's emergence from his self-incurred immaturity. Immaturity is the inability to use one's own understanding without the guidance of another. […] If I have a book to have understanding in place of me, a spiritual adviser to have a conscience for me, a doctor to judge my diet for me, and so on, I need not make any efforts at all. I need not think, so long as I can pay; others will soon enough take the tiresome job over for me. The guardians who have kindly taken upon themselves the work of supervision will soon see to it that by far the largest part of mankind (including the entire fair sex) should consider the step forward to maturity not only as difficult but also as highly dangerous.

In this quote, one can sense what the age of enlightenment intended by education. Since present ways of teaching leave the student's perception of self at the side of the educational system, the above-mentioned problems can be understood as a lack of enlightenment in mathematics teacher education.

What alternative ways and models are there to change the sense of self and the self-esteem of students in order for them to become more autonomous individuals?

COMMUNITIES OF PRACTICE AND A GROWING MATURITY

In order to become autonomous, mature individuals the students need to experience a sense of self in which their own personal values for their development arise, ones that are supported by a recognition of other like-minded people with similar developing interests – backed by universal values and rules. In that way, they can actualize themselves and determine their own actions.

Hence, we need a course that allows university teachers and students to develop a community of practice, which fosters their development as far as orientation, maturity, autonomy, emancipation, responsibility, self-actualization and self-determination are concerned. To this end, we withdraw our guidance in small steps and replace it by development through progress in the community. Such development of value systems has been described in the discourse on communities of practice. For instance in (Wenger, 2002; chapter 8), we find several procedures for measuring and managing the value creation of a community or network. Such value creation methods are nowadays widely used in management. When we compare the five cycles of value creation, we discover some similarity between this procedure and the five basic questions for the preparation of lessons as Klafki (1963) formulated them. We use the value assessment framework as tool to structure the seminar and its development. A constant reframing and reconsideration of how success, appreciation and development are defined characterize the framework.

A SEMINAR ON HISTORICAL EXCERPTS FROM SCHOOL BOOKS

Historical excerpts from schoolbooks form the basis for an activity in the framework of the seminar. The goal is to undertake small-step lesson planning, starting from the "historical" excerpt. How can such an existing historical excerpt be augmented to curriculum-relevant teaching that serves the *Bildung* of the students? In Germany, there are a handful of schoolbook series that are used extensively in school (Rezat (2010), Otte (1981)). The historical insertions in these books are all of a similar kind. Since these historical references stem from books that teachers use in their daily teaching at schools, they constitute a link of this activity with the practice. Indeed, when the future teacher finds her or himself teaching with the help of such a schoolbook later, it might be an occasion to unfold the learned attitude again – together with students and colleagues. In addition, it would be possible to get teachers from school involved in the project. Therefore, the seminar at the university already has the goal of letting the participants find like-minded people, who are also interested in history of maths and the development of mathematical contents.

Designing the seminar and also tackling the aforementioned problems in self-esteem and predominance of normative results, means starting with sufficiently open but concrete tasks and leaving a lot of time for group discussions: "A key element of designing for value is to encourage community members to be explicit about the value of the community throughout its lifetime. Initially, the purpose of such discussion is more to raise awareness than to collect data, since the impact of the community typically takes some time to be felt." (Wenger et al., 2002; p. 60).

A motive of development and support for a community of practice uniting student maths teachers, maths teachers, maths educators, maths textbook authors, historians, educators, other social science people and workers in further education (teacher development) leads to various activities. This motive defines activities and possible actions during the running seminar as well as long term planning as the:

- organisation of student teaching to apply the developed material,
- establishment of connections to textbook authors,
- development of an internet page with additional materials to the historical excerpts,
- linking of related existing internet sources with the materials of the seminar,
- development of activities for teacher in-service training on the basis of the additional materials to the historical excerpts and some aspects of the use of history in maths classes.

For our project in Germany the enormous influence of maths textbooks on teaching – especially at the beginning of the career – is essential. In other countries and periods, this might be different. For the development of a community of practice in the spirit we are aiming at, it is important that the joint activities generate joint creations, have a context that is relevant to the participants, are related to their personal experiences and manageable by any member (the excerpts are small – about one page). An important step in transforming the seminar into a community of practice is the emergence and display of immediate values and potential values of the community. Having this in mind, monitoring, structuring and back up of the first group's discussions are important.

The students work in groups on one excerpt. They choose the topic and the material from the books on different grounds, for example:

- individual historical or mathematical interests,
- relations to other school subjects,
- experiences from their practical lessons or teaching in school,
- experiences in tutoring students,
- the wish to be in a team with a best friend.

The activities and interactions between members of the seminar in the group discussions, the choice of the material and plans to unfold it are important for the immediate and potential value of the further community.

Intentions of the discussions are, for example:

- for students to get to know their biases and preferences,
- to start a general discussion about the use of textbooks and the particular design of the textbooks they will deal with,

- the realising of interrelations between topics studied in history of mathematics courses,
- the introduction of existing materials related to the excerpts,
- the repetition of basics in lesson planning and textbook use.

According to Wenger et al. (2002), one of the tasks of the seminar leader is to understand which potentials and *knowledge capital* of the community can be put into use. It may be helpful to give introductory presentations to support structure and to display possible joint activities.

When dealing with a schoolbook there are essentially three perspectives that one can take: the high school student's perspective, the teacher's perspective, or the perspective of the schoolbook author. While being used to the first perspective, the students usually struggle with the two more mature ones. Here we see the major role of communities of practice.

The textbook analysis starts from the perspective of a student, e.g. reading of the chosen excerpts, solving related problems, clarifying prerequisites and presenting the results. The teacher's perspective comes with learning objectives, reflection on the assumed knowledge, time management, and representation of the solutions of the problems. The perspective of an author appears in questions about the way history is used, the accuracy of the presentation of the historical fact, the design of the historical excerpt and the learning objectives of the involvement of history.

The three perspectives are understood as levels of awareness (Mason, 1998). Cooperation of the students is organised by grouping around interests in mathematics, history or interdisciplinary questions related to concept development. For the analysis of the mathematical textbook students can take a historical perspective, asking questions related to the story of a mathematical problem, the history of an idea, the story of a mathematical area, the biography of a mathematician, the story of an institution, the history of a concept,

Questions arising from a mathematical perspective are:

- How do modern notations and representations differ from historical ones?
- Which problems led to the imposition of a term?
- What is the mathematical statement of a historical mathematical text source?

The perspective involving reflection on historical, mathematical and cognitive development is the most complex and advanced. It includes theoretical frames for concept development with historical references; that is to say, a historical genetic approach, a hermeneutic approach, mathematical awareness, use of history as a mathematical tool, 'Whiggish' approaches or cognitive genetic and historical genetic approaches.

Student work is organised in groups attached to an excerpt. There can be several groups working on one historical textbook excerpt. The joint product of the group activities consists of a joint essay and short lesson plan related to different perspectives, from a historical and from a mathematical point of view. Let us consider two examples.

Example from Geometry – Reproductions , Excerpts from historical sources

Here, we consider a reproduction of a small part of a small historical source. We look (Figure 1) at the *historical proof* of the irrationality √2 from Euclid, which one can find in most textbooks in a similar way:

Examples	Proof that $\sqrt{2}$ is an irrational number. *Assertion*: $\sqrt{2}$ is irrational. *Proof by contradiction*: Assume the opposite is true.	
The proof on the right is the eldest historically handed down proof for the irrationality of $\sqrt{2}$. At the same time, it is an especially beautiful example of an indirect proof.  	Consequences:	Explanation:
	(1) $\sqrt{2}$ is irrational.	
	(2) $\sqrt{2} = \frac{p}{q}$	Can $\sqrt{2}$ be represented as simplified fraction $\frac{p}{q}$?
	(3) $2 = \frac{p^2}{q^2}$	squaring of the equation
	(4) $p^2 = 2q^2$	resolving to p^2
	(5) p^2 is even	property of even numbers
	(6) p is odd	property of odd numbers
	(7) $p = 2n$	p is even, hence divisible by 2
	(8) $p^2 = 4n^2$	squaring of (7)
	(9) $4n^2 = 2q^2$	inserting (8) in equation (4)
	(10) $2n^2 = q^2$	dividing the equation (9) by 2
	(11) q^2 is even	property of even numbers
	(12) q is odd	property of odd numbers
	(13) $q = 2m$	q is even, hence divisble by 2.
	(14) $\frac{p}{q}$ is not a completely simplified fraction	p and q are even, so both are divisible by 2 .
This is a proof by contradiction to the assumption that $\sqrt{2}$ is representable by a completely simplified fraction. Hence, $\sqrt{2}$ is irrational.		

Figure 1: Historical excerpt from a German schoolbook (Neue Wege, 9. Schuljahr)

First students will study the proof from the perspective of a high school student. Corresponding mathematical questions from their perspective could concern the logic of the indirect proof, the logic of the arguments, proof of the used arguments, mathematical terms and notations.

From a historical perspective, it is natural to search for the primary source and to compare it with the reproduction. In this case, it leads to an interesting search for a translation, containing the mentioned proof. At this point, the discussion can be supported by translations with commentaries introducing the students to historical questions that are related to translations and the choice of documents we are referring to when citing Euclid.

From the perspective of a teacher, the students would discuss whether the presentation helps to support mathematical understanding, what additional materials with enactive and iconic presentations can be used, and how to evaluate the understanding of the proof. Historical questions could deal with the use of original sources, with additional materials about Greek mathematics, and interdisciplinary learning environments relating to history, mathematics and philosophy.

Taking the perspective of a textbook author one could start with the question of whether history in this excerpt is used as a tool or as an object (Jankvist, 2009). Looking at the presentation and the use of modern notations can lead to questions related to *Whiggish* (Fried, 2001) presentations and their justifications. Another aspect is the use of geometrical concepts in Euclid's books and their algebraization in modern school mathematics.

To support group discussions, the tutor could give introductions to Greek mathematics, an overview of Euclid's *Elements* or an introduction to Geometric Algebra. Both translations and secondary literature on Euclid are easily accessible to the students. In this case, one would initially not provide reading material but rather help students to organise and structure the found sources.

An Example from Analysis

We now have an example of another kind. The textbook authors give a historical overview over a longer period. This excerpt at hand starts with general remarks about the roots of mathematics and analysis. The first mentioned mathematicians reach from Archimedes to Riemann. The excerpt ends with general remarks on the contemporary role of analysis and its central place in the study of mathematics. The subsequent questions are challenging for high school students and for the student teachers in the seminar too. In the case of an overview, a restriction on a well-selected aspect is helpful. In the case of the development of calculus Barrow-Green (2008) gives a wonderful example of how to illustrate development.

Excursion

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After these preparations, in the 17th century, the Englishman Isaac Newton and the German Gottfried Wilhelm Leibniz independently laid the foundation for analysis with infinitesimal calculus.

Newton assumed variable magnitudes to be time-dependent and called them “fluents” (flowing ones). By the derivative with respect to time he denoted their instantaneous velocities (“derivative”), which he called “fluxions” and marked them with a dot (e.g. $\dot{x}$).

Newton calculated the fluxions by limit considerations. Since such a practice did not fit to his own methodological ideals, at first he did not publish his results, but just mentioned them indirectly while arguing with time-independent geometrical magnitudes.

In that way, it happened that the Germen Gottfried Wilhelm Leibniz developed about ten years later his own theory for the notion of derivative. Leibniz regarded a curve as an “infinity-gon”, such that a tangent would intersect the curve in an infinitely small line segment. Here he built, amongst other things, on the insights of Cavalieri. Leibniz introduced the notion of “differentials”, which brought forth the notion of “differential calculus”. The quarrel between Newton and Leibniz, about which one of them would first have discovered the notion of derivative, has found its way as priority dispute into history of mathematics.

In the course of time, analysis was first further developed without really substantiated foundations. Only in the 19th century could one work with it in a way that fits today’s standards, for only since then have notions like function, limit or integral been clarified precisely. To this end, the mathematicians Joseph Louis Lagrange (1736-1813), Augustin Louis Cauchy (1789-1857), Karl Weierstraß (1815-1879), Carl Friedrich Gauß (1777-1855) and Richard Dedekind (1831-1916) contributed in crucial ways.

The integral, in the form it is taught nowadays at grammar schools, goes back to the German mathematician Georg Friedrich Bernhard Riemann. Riemann determined the area that is bounded by the x-axes and the graph of a function by the help of easy to calculate areas of rectangles. The idea of the so-called “Riemann integral” was later further developed by the French mathematician Henri Léon Lebesgue (1875-1941).

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Figure 2: Historical excerpt from a schoolbook (Lambacher Schweitzer, S. 165-166)

For the development of the perspective of a teacher or even a textbook author additional help by the tutor is necessary.

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Prepare a presentation about the history of analysis. To this end, read up on more contributions of the mathematicians that are introduced in the excursion. Take the following points into account:

- Which mathematician is credited with the discovery of the exhaustion method?

- How did Galilei determine the velocity resp. the acceleration of a ball rolling down an inclined plane?

- What is understood by the theorem of Cavalieri?

- Explicate Newton and Leibniz's approach to the notion of derivative, giving examples.

Figure 3: Exercise related to the historical excerpt in Figure 1

The excerpt offers at several places the opportunity to go deeper into history. This single excerpt can already supply the seminar with questions and aspects to be explored for many semesters. For our example we restrict ourselves to just one of them.

RESUMÉ

We are aware that the development of a community of practice of mathematics teachers and mathematics educators interested in history is a long-term task.

At the present economising of university life and the strong dominance of normative value systems we consider this experiment nevertheless particularly important.

NOTES

1. For learning mathematics via historical sources see for instance: www.uni-due.de/didmath/ag_jahnke/historische, www.fransvanschooten.nl, www.cs.nmsu.edu/historical-projects/ or www.pageaboutme.mathsisgoodforyou.com/.

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